KYUSHU-HET-122
KUNS-2233

Geometry-free neutrino masses in curved spacetime

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(October, 2009)

Abstract

The seesaw-induced neutrino mass is discussed in a generic class of curved spacetime, including the flat and warped extra dimensions. For Majorana masses in the bulk and on the boundary, the exact forms of seesaw-induced masses are derived by using the Kaluza-Klein mode expansion and the lepton number violating correlator for bulk fermion. It is found that the neutrino mass is determined without the knowledge of wave functions and whole background geometry when the metric factor is fixed on the boundary, e.g. by solving the hierarchy problem.

1 Introduction

Over the past decade, theories with extra dimensions have attracted great attention as feasible paradigm to understand the unresolved problems in the Standard Model (SM). For example, the four-dimensional Planck scale becomes effective, which is made out of the fundamental scale of higher-dimensional gravity and the large volume size of extra space by which the matter-gravity coupling is weakened [1]. The localized gravity with the warped metric [2] also provides a framework for solving the gauge hierarchy by small overlap between matter and gravitational fields.

An interesting aspect of higher-dimensional theory is the interplay with the neutrino physics. As in the same way that the flux is diluted in the extra space, the low-energy neutrino mass receives a volume suppression if right-handed neutrinos live in the extra dimensions [3]. It has also been discussed that the localization of bulk fermions in the warped extra dimension produces tiny neutrino masses [4]. Such a connection between the neutrino physics and extra dimensions is a subject of great interests to particle physics.

In this letter, we explore the bulk Majorana neutrinos in a generic type of non-factorizable geometry, including the flat and warped extra dimensions. According to the location of lepton-number violation, we separately consider two types of Majorana mass terms consistent with the Lorentz invariance: (i) in the bulk and (ii) on the Planck brane. (The TeV-brane mass term is a straightforward application.) While the exact form of mass spectrum and wave functions cannot be found for bulk fermions, the low-energy effective mass for left-handed neutrinos is obtained analytically.

2 Majorana mass on the Planck brane

Throughout this letter, we consider the five-dimensional theory on the gravitational background with the following non-factorizable form

$$ds^2 \;=\; g_{MN}dx^M dx^N \;=\; \rho^{-2}(y)\,\eta_{ab}dx^a dx^b - dy^2, \tag{2.1}$$

where η_{ab} is the four-dimensional Minkowski metric. The fifth dimension has two orbifold fixed points at $y = 0$ and $y = L$. A phenomenologically interesting example is the anti de-Sitter (AdS) space where the metric factor $\rho(y)$ is given by $e^{k|y|}$ (k is the AdS curvature). In this case, the physical scales for $y = 0$ and $y = L$ boundaries are differently set to

the Planck and TeV scales with the warp factor e^{-kL}, and they are referred to as the Planck and TeV branes. An index of $\mathcal{O}(10)$, namely $kL \simeq 37$, is sufficient for obtaining the electroweak/Planck mass hierarchy.

The SM fields, especially the left-handed neutrinos N and the Higgs field H are assumed to be localized at $y = L$. The right-handed neutrinos are introduced as bulk Dirac fermions $\Psi(x, y)$ which obey the boundary conditions $\Psi(x, -y) = \gamma_5\Psi(x, y)$ and $\Psi(x, L - y) = \gamma_5\Psi(x, L + y)$. Let us consider the Majorana mass on the Planck brane. The Planck-brane Majorana mass on the warped AdS$_5$ background has been discussed in various setups [5]. In the general non-factorizable geometry (2.1), the Lagrangian relevant for neutrino physics is given by

$$\begin{aligned}\mathcal{L} \;\; = \;\; \sqrt{g}\Big[& i\overline{\Psi}\Gamma^M D_M\Psi - m_d\theta(y)\overline{\Psi}\Psi - \Big(\frac{1}{2}M\overline{\Psi^c}\Psi + \text{h.c.}\Big)\delta(y) \\ & + \big[\, i\overline{N}\gamma^\mu\partial_\mu N - \rho\left(m\overline{\Psi}N + \text{h.c.}\right)\big]\,\delta(y-L)\Big],\end{aligned} \tag{2.2}$$

where D_M is the covariant derivative which includes the spin connection, and the gamma matrices are related as $\Gamma^\mu = \gamma^\mu$ and $\Gamma^y = i\gamma_5$. The step function $\theta(y)$ is needed since the mass operator $\overline{\Psi}\Psi$ is odd under the reflection parities with respect to $y = 0$ and $y = L$. The charge-conjugated spinor Ψ^c is defined as $\Psi^c = \Gamma^3\Gamma^1\overline{\Psi}^{\mathrm{T}}$ such that it becomes Lorentz covariant in five dimensions. The parameter m means the electroweak-breaking Dirac mass given by the vacuum expectation value $\langle H\rangle$. In the above Lagrangian, we have rescaled the Higgs field so that its kinetic term becomes canonical. That leads to the ρ factor in the boundary mass term, and hence m is regarded as a parameter of the electroweak scale.

The low-energy effective theory for neutrino masses is deduced by usual Kaluza-Klein (KK) expansion. After rescaling Ψ and N so that their kinetic terms become canonical, we expand the right-handed neutrino fields as

$$\Psi(x, y) \;\; = \;\; \begin{pmatrix} \sum_n \chi_R^n(y)\psi_R^n(x) \\ \sum_n \chi_L^n(y)\psi_L^n(x) \end{pmatrix}, \tag{2.3}$$

with the wave functions $\chi^n_{R,L}(y)$ which satisfy the equations of motion

$$\Big(\partial_y + m_d\theta(y) - \frac{\partial_y\rho}{2\rho}\Big)\chi_R^n \;\; = \;\; +M_{K_n}\rho(y)\chi_L^n, \tag{2.4}$$

$$\Big(\partial_y - m_d\theta(y) - \frac{\partial_y\rho}{2\rho}\Big)\chi_L^n \;\; = \;\; -M_{K_n}\rho(y)\chi_R^n, \tag{2.5}$$

where M_{K_n} represent the KK mass eigenvalues. The non-trivial solution for the zero mode is given by

$$\chi_R^0(y) \;=\; A\, e^{-m_d|y|} \rho^{\frac{1}{2}}(y)\,. \tag{2.6}$$

The normalization constant A is fixed by the condition $\int_0^L dy\, (\chi_R^0)^2 = 1$. The localization profile of the zero mode is fixed by the bulk Dirac mass m_d and the metric factor.

Substituting the KK-mode expansion into (2.2) and integrating it over the extra space, we obtain the four-dimensional effective masses for the normalized Majorana fermions:

$$\mathcal{M} = \left(\begin{array}{c|c} 0 & \mathcal{M}_D^{\rm T} \\ \hline \mathcal{M}_D & \mathcal{M}_H \\ & \end{array}\right) = \left(\begin{array}{c|cccc} & m_0^{\rm T} & m_1^{\rm T} & & \cdots \\ \hline m_0 & -M_{R_{00}}^* & -M_{R_{01}}^* & & \cdots \\ m_1 & -M_{R_{10}}^* & -M_{R_{11}}^* & M_{K_1} & \cdots \\ & & M_{K_1} & & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{array}\right), \quad \mathcal{N} = \begin{pmatrix} \nu_L \\ \epsilon\psi_R^{0\,*} \\ \epsilon\psi_R^{1\,*} \\ \psi_L^1 \\ \vdots \end{pmatrix}, \tag{2.7}$$

$$M_{R_{nm}} \;=\; \rho^{-1}(0)\, \chi_R^{n\,{\rm T}}(0) M \chi_R^m(0), \qquad m_n \;=\; \chi_R^{n\,\dagger}(L)\, m, \tag{2.8}$$

where ν_L is the left-handed neutrinos in the two-component notation: $N(x) = \binom{0}{\nu_L}$. Besides the upper-left entry, the vanishing elements in the mass matrix $\mathcal{M}$ come from the Dirichlet conditions $\chi_L^n(0) = \chi_L^n(L) = 0$.

The low-energy mass spectrum is obtained by the diagonalization of $\mathcal{M}$. Under the assumption $m_n \ll M_{R_{nm}}$, the Majorana mass matrix for the left-handed neutrinos is approximated as $M_\nu = -\mathcal{M}_D^{\rm T} \mathcal{M}_H^{-1} \mathcal{M}_D$. The inverse of the heavy-sector infinite matrix $\mathcal{M}_H$ is found

$$\mathcal{M}_H^{-1} \;=\; \begin{pmatrix} \frac{-1}{M_{R_{00}}^*} & & \frac{M_{R_{01}}^*}{M_{R_{00}}^*}\frac{-1}{M_{K_1}} & & \frac{M_{R_{02}}^*}{M_{R_{00}}^*}\frac{-1}{M_{K_2}} & \cdots \\ & & \frac{1}{M_{K_1}} & & & \cdots \\ \frac{M_{R_{01}}^*}{M_{R_{00}}^*}\frac{-1}{M_{K_1}} & \frac{1}{M_{K_1}} & & & & \cdots \\ & & & & \frac{1}{M_{K_2}} & \cdots \\ \frac{M_{R_{02}}^*}{M_{R_{00}}^*}\frac{-1}{M_{K_2}} & & & \frac{1}{M_{K_2}} & & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}. \tag{2.9}$$

Multiplying $\mathcal{M}_D$ from both sides, we find that all the KK-mode contributions vanish, except for the zero-mode one:

$$M_\nu \;=\; \frac{m_0^{\rm T} m_0}{M_{R_{00}}^*} \;=\; e^{-2m_dL} \rho(L)\, \frac{m^{\rm T} m}{M^*}. \tag{2.10}$$

The seesaw-induced mass has been studied by truncating the large matrix $\mathcal{M}$ to include finite numbers of KK modes. However the infinite-dimensional matrix (2.9) shows that the exact form of KK-induced Majorana masses is obtained without the knowledge of mass spectrum and wave functions of KK-excited modes, which are generally complicated to perform the seesaw operation.

The effective mass M_ν involves only the zero-mode piece and the other ones are cancelled out. This is because the lepton number is violated only in the zero-mode sector, which is clearly seen in the matrix (2.9) whose elements conserve the lepton number except for the zero-mode one, $-1/M^*_{R_{00}}$. One can also see this by making the mixed states between ψ^0_R and ψ^n_R with the rotation angles $\tan\theta_n = \chi^n_R(0)/\chi^0_R(0)$. With this change of basis, the lepton-number violating terms in the heavy-sector matrix $\mathcal{M}_H$ are all erased out besides the zero-mode entry. In the case of localized Majorana masses, therefore, only the zero mode ψ^0_R takes part in the seesaw mechanism. It should be noted that ψ^0_R is not a mass eigenstate in four-dimensional viewpoint. The real "zero mode", which would be observed in future particle experiments, is a linear combination of KK modes.

The suppression of neutrino mass (2.10) to the eV range is achieved with appropriate bulk masses and gravitational background. The zero modes of bulk neutrinos should be localized towards the TeV brane, which reduces the Planck-brane Majorana mass to an intermediate seesaw scale. If the metric factor $\rho(L)$ is used to resolve the gauge hierarchy, $M\rho^{-1}(L)$ is around or smaller than the TeV scale.* Therefore the wave function factor with a (positive) non-vanishing m_d is suitable for having tiny neutrino masses. To be more concrete, let us consider the warped AdS$_5$ metric ($\rho(y) = e^{k|y|}$) and parameterize the bulk Dirac and Majorana masses as $m_d = c_d k$ and $M = c_M k$. From (2.10), it follows that a coefficient $c_d \simeq 0.38$ produces $\mathcal{O}(0.1)$ eV neutrino mass for $c_M = 1$. Around a suppressed value $c_M \sim 10^{-4}$, the coefficient c_d reaches the de-localized limit $c_d = 0.5$ and the zero mode starts to be localized towards the Planck brane.

Another way to realize a tiny Majorana neutrino mass is to suppress the boundary Dirac mass (i.e. the Yukawa coupling). That is related to the charged-lepton sector on the TeV brane. One may apply some mechanisms which have been proposed in four or higher-dimensional theory to solve the fermion mass problem, e.g. with an abelian flavor

*This conclusion can be changed by considering a bulk Higgs field and/or a small Higgs mass, which is stabilized by other dynamics such as supersymmetry.

symmetry *à la* Froggatt and Nielsen [6]. A different approach is to assume that the left-handed neutrinos (lepton doublets) and/or the right-handed charged leptons also access the extra dimension. For an example where the right-handed charged leptons propagate in the warped extra dimension, the bulk Dirac masses $(m_{d_e}, m_{d_\mu}, m_{d_\tau}) = (0.842, 0.688, 0.603)k$ reproduce the observed charged-lepton masses for unit Yukawa couplings. If the lepton doublets also feel the extra dimension, the zero modes of right-handed neutrinos should get closer to the TeV brane for compensating the suppression by the left-handed neutrinos.

3 Majorana mass in the bulk

Another interesting possibility for neutrino physics, though less considered, is to put the Majorana mass in the five-dimensional bulk. As in the previous section, the left-handed neutrinos N and the Higgs field H are assumed to reside on the TeV brane. In addition, we introduce the five-dimensional right-handed neutrinos Ψ with the Majorana mass term in the bulk. The Lagrangian on the general non-factorizable background (2.1) is given by

$$\begin{aligned} \mathcal{L} \;=\; & \sqrt{g}\Big[\, i\overline{\Psi}\Gamma^M D_M\Psi - m_d\theta(y)\overline{\Psi}\Psi - \Big(\frac{1}{2}M\overline{\Psi^c}\Psi + \text{h.c.}\Big) \\ & \qquad\qquad + \big[\, i\overline{N}\gamma^\mu\partial_\mu N - \rho\left(m\overline{\Psi}N + \text{h.c.}\right)\big]\,\delta(y-L)\Big]. \end{aligned} \tag{3.1}$$

The bulk fields Ψ are supposed to obey the boundary conditions $\Psi(x,-y) = \gamma_5\Psi(x,y)$ and $\Psi(x,L-y) = \gamma_5\Psi(x,L+y)$ as in the previous section. Other types of boundary conditions will be discussed later. The bulk Majorana mass in the warped AdS_5 geometry has been mentioned with a different type of compactification [7]. In this letter, we do not consider the Lorentz-violating Majorana mass of the form $\overline{\Psi^c}\gamma_5\Psi$, while often studied in the literature.

For the bulk Majorana mass, it is not easy to find an analytic form of the seesaw-induced mass by the KK expansion method. With the eigenfunctions obeying (2.4) and (2.5), the neutrino mass matrix $\mathcal{M}$, which corresponds to (2.7), becomes

$$\mathcal{M} \;=\; \left(\begin{array}{c|c} 0 & \mathcal{M}_D^{\mathrm{T}} \\ \hline \mathcal{M}_D & \mathcal{M}_H \end{array}\right) \;=\; \left(\begin{array}{c|cccc} & m_0^{\mathrm{T}} & m_1^{\mathrm{T}} & & \cdots \\ \hline m_0 & -M_{R_{00}}^* & -M_{R_{01}}^* & M_{L_{01}} & \cdots \\ m_1 & -M_{R_{10}}^* & -M_{R_{11}}^* & M_{K_1} & \cdots \\ & M_{L_{10}} & M_{K_1} & M_{L_{11}} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{array}\right). \tag{3.2}$$

The KK-mode Majorana masses are determined by

$$M_{R_{nm}} \;=\; \int_0^L \! dy\, \rho^{-1} \chi_R^n(y)^{\mathrm{T}} M \chi_R^m(y), \qquad M_{L_{nm}} \;=\; \int_0^L \! dy\, \rho^{-1} \chi_L^n(y)^{\mathrm{T}} M \chi_L^m(y). \tag{3.3}$$

The matrix $\mathcal{M}$ is too complicated for the seesaw integration to be performed: the heavy-sector matrix $\mathcal{M}_H$ cannot be obtained explicitly due to intricate wave functions. Moreover, unlike the previous case, there is no trivially-vanishing element in $\mathcal{M}_H$ and it seems hard to write down its inverse $\mathcal{M}_H^{-1}$ and to evaluate the seesaw formula.

As an alternative to the KK-mode expansion, the propagator method for bulk fermion is suitable for calculating low-energy neutrino masses. For the present purpose, it is convenient to make a non-canonical rescaling $\Psi \to \rho^2 \Psi$ and the Lagrangian is then rewritten as

$$\mathcal{L} \;=\; \frac{1}{2}\overline{\Phi} D \Phi + \sqrt{g}\left[\, i\overline{N}\gamma^\mu \partial_\mu N - \rho^3\big(m\overline{\Psi}N + \text{h.c.}\big)\,\right]\delta(y-L), \tag{3.4}$$

where

$$D \;=\; \begin{pmatrix} i\rho\not{\partial} - \gamma_5\partial_y - m_d\theta(y) & -M^* \\ -M & i\rho\not{\partial} - \gamma_5\partial_y + m_d\theta(y) \end{pmatrix}, \qquad \Phi \;=\; \begin{pmatrix} \Psi \\ \Psi^c \end{pmatrix}. \tag{3.5}$$

The lepton number violating part of the two-point function is extracted from the inverse of D. Regarding the TeV-brane mass as a perturbation, we find the effective Majorana mass for the canonically-normalized left-handed neutrinos,

$$M_\nu \;=\; \rho(L)\, m^{\mathrm{T}} \langle \epsilon \nu_R^*(p,L) \nu_R^\dagger(p,L)\rangle\big|_{p=0}\, m, \tag{3.6}$$

where ν_R is the right-handed component of (rescaled) bulk spinor Ψ, and p is the momentum in the four-dimensional Minkowski spacetime. If one substitutes the KK-mode expansion $\nu_R = \sum_n \rho^{\frac{-1}{2}} \chi_R^n(y)\psi_R^n(p)$ into the formula ($\rho^{\frac{-1}{2}}$ implies the above rescaling and the canonical normalization of ψ_R^n),

$$\begin{aligned} \rho(L)\, m^{\mathrm{T}} \langle \epsilon \nu_R^*(p,L)\nu_R^\dagger(p,L)\rangle m \;&=\; \sum_{k,n} m^{\mathrm{T}} \chi_R^k(L)^* \langle \epsilon \psi_R^k(p)^* \psi_R^n(p)^\dagger \rangle \chi_R^n(L)^\dagger m \\ &=\; \sum_{k,n} m_k^{\mathrm{T}} \left(\frac{\mathcal{M}_H^*}{p^2 - \mathcal{M}_H^*\mathcal{M}_H}\right)_{R_{kn}} m_n\,, \end{aligned} \tag{3.7}$$

where R_{kn} means the matrix element for the KK right-handed neutrinos $\psi_R^{k,n}$. The last line is the summation over all heavy-mode contributions, just as performed in the previous section. With the propagator at hand, therefore, one needs neither to integrate over the extra space nor to diagonalize the infinite neutrino matrix.

It is generally difficult to find the bulk Majorana propagator. However in view of the seesaw scheme, we only need the low-energy limit $p \to 0$.† It is seen from (3.5) that, in the low-energy limit, the metric factor ρ vanishes away from the problem and the propagator is found to have the same form as in the flat extra dimension. Notice that the low-energy limit $p \to 0$ is allowed if the solution is not singular in this limit. For neutrino physics, the Majorana mass lifts the chiral zero mode of bulk fermion up to the heavy sector, and the propagator does not have a pole at $p = 0$.

By solving the inverse of D without $\partial\!\!\!/$ parts and setting the boundary conditions $\Psi(x,-y) = \gamma_5\Psi(x,y)$ and $\Psi(x,L-y) = \gamma_5\Psi(x,L+y)$, we find the lepton number violating correlator

$$\begin{aligned}\langle\epsilon\nu_R^*(0,y)\nu_R^\dagger(0,y')\rangle &= \frac{M}{(q^2-m_d^2)q\sinh(qL)}\Big[q\cosh(qy_<) - m_d\sinh(qy_<)\Big] \\ &\qquad \times\Big[q\cosh(qy_> - qL) - m_d\sinh(qy_> - qL)\Big], \quad (3.8)\end{aligned}$$

where $q^2 = m_d^2 + |M|^2$ and $y_<$ ($y_>$) stands for the lesser (greater) of y and y'. From the formula (3.6), the Majorana mass is induced for left-handed neutrinos in the general non-factorizable geometry:

$$M_\nu \;=\; \rho(L)\,\frac{q\cosh(qL) - m_d\sinh(qL)}{\sinh(qL)}\,\frac{m^{\rm T}m}{M^*}. \qquad (3.9)$$

The typical heavy-mode scale is played by the bulk Majorana mass M or the KK scale. Let us take $m_d = 0$ for simplicity. For a smaller value of Majorana mass, $qL \ll 1$, the neutrino mass is approximated as $M_\nu \simeq \frac{\rho(L)}{L}\frac{m^2}{M^*}$. In the KK-mode picture, the contribution from low-lying states, which is governed by the effective Majorana mass $M\rho(L)^{-1}$, dominates the seesaw-induced mass. In the opposite limit $qL \gg 1$, the neutrino mass follows $M_\nu \simeq \frac{\rho(L)}{L}\frac{m^2}{(1/L)}$, and the KK scale $\frac{1}{L}\rho^{-1}(L)$ plays a role of the seesaw denominator.

If the metric factor $\rho(L)$ is used to resolve the gauge hierarchy, the heavy-mode scales $M\rho^{-1}(L)$, $\rho^{-1}(L)/L$ are around or smaller than TeV. In the present setup, there are several ways to reproduce a proper scale of neutrino mass. A direct approach is to consider a bulk Majorana mass of an intermediate scale. For instance, in the case that $M \ll m_d$ and $qL \gg 1$, the neutrino mass is given by $M_\nu \simeq \rho(L)m^2M/m_d$ and

†The zero-momentum limit is not actually required, but $\rho(y)p$ should be smaller than the fundamental scale at any point in the bulk for the following procedure being valid.

therefore the effective heavy-mode scale is enhanced by m_d/M. Thus a small lepton number violation, $M/m_d \sim 10^{-10}$, leads to an eV-order neutrino mass.

A different type of boundary conditions for the bulk neutrino $\Psi(x,y)$ is a new interesting possibility for neutrino phenomenology. Under the conditions $\Psi(x,-y) = -\gamma_5\Psi(x,y)$ and $\Psi(x,L-y) = \gamma_5\Psi(x,L+y)$, the lepton number violating part of the correlator becomes in the low-energy regime

$$\begin{aligned}\langle \epsilon\nu_R^*(0,y)\nu_R^\dagger(0,y')\rangle &= \frac{M\sinh(qy_<)}{q\Big[q\cosh(qL)+m_d\sinh(qL)\Big]} \\ &\qquad\times\Big[q\cosh(qy_>-qL)-m_d\sinh(qy_>-qL)\Big], \qquad (3.10)\end{aligned}$$

where the definitions of q and $y_{\lessgtr}$ are the same as in (3.8). With the propagator at hand, the neutrino mass reads

$$M_\nu \;=\; \rho(L)\,\frac{\sinh(qL)}{q\cosh(qL)+m_d\sinh(qL)}\,m^{\mathrm{T}}Mm\,. \qquad (3.11)$$

This is the exact expression for taking into account of all KK-mode contributions in the general non-factorizable geometry. Unlike the usual seesaw mechanism, the heavy Majorana mass M appears in the numerator of (3.11). In the limit $qL \gg 1$, the neutrino mass becomes $M_\nu \simeq \rho(L)\big(\frac{M}{q+m_d}\big)m^2$, which is equivalent to M_ν for the previous boundary condition. This is because, in the large-size limit of extra dimension $qL \gg 1$, the difference of boundary conditions at $y=0$ (for the right-handed component) is irrelevant to the physics at another boundary where the left-handed neutrinos reside. On the other hand, for the opposite limit $qL \ll 1$, the neutrino mass M_ν is approximated as $M_\nu \simeq \rho(L)LMm^2$. In the KK-mode picture, the low-energy spectrum from Ψ has no chiral zero mode and consists of vector-like KK fermions which are perturbed by small Majorana masses. Therefore the seesaw-induced mass from these states is proportional to M, not inverse-proportional as in the usual seesaw mechanism. Thus a small ratio of M to the compactification scale $1/L$ serves as a suppression factor for the neutrino mass: for instance, an eV-order M_ν is reproduced by $ML \sim 10^{-10}$.

As mentioned previously, the extension of left-handed neutrinos (lepton doublets) to the extra dimension is also a possible way to realize a tiny mass scale M_ν by reducing the boundary Dirac mass m (i.e. the Yukawa coupling) to a smaller value than the electroweak scale. The suppression with the wave functions of left-handed neutrinos cannot be arbitrarily strong since it also brings down the charged-lepton mass scale.

To estimate to what extent their wave functions can suppress the neutrino mass, let us assume the right-handed tau to reside on the TeV brane. In this case, the neutrino mass M_ν is suppressed by the factor of $(m_\tau/\mathrm{TeV})^2 \sim 10^{-6}$.

4 Summary and discussion

We have discussed the seesaw mechanism with bulk and boundary Majorana mass terms in the general non-factorizable geometry, referring to the warped AdS_5 metric as a typical example. We have derived the exact seesaw-induced masses by the Kaluza-Klein expansion and the propagator method, and presented the results in analytic forms which make it straightforward to analyze what types of effects are involved in the induced neutrino masses. The details of wave functions and background geometry are irrelevant to the light neutrino mass when the metric factor is fixed, e.g. by solving the gauge hierarchy problem. The observed tiny mass scale of left-handed neutrinos is reproduced in both setups with small lepton number violation or other effects.

It is easy to include in the present framework the three generation fermions and their mixture. The flavor structure is introduced with masses and couplings in the generation space, which would be controlled by some fundamental dynamics to be specified. Within the higher-dimensional theory, the observed neutrino mixing is obtained, for example, by flavor symmetry and its breaking by orbifold boundary conditions imposed on bulk fields [8], where non-abelian discrete flavor symmetry is adopted and the tri-bimaximal generation mixing [9] is realized as a direct consequence of the theory. On the other hand, it is often discussed in flavor theory that different generations have different position profiles in the extra dimensions [10] and, if bulk right-handed neutrinos included, they connect up these generations across the extra dimensions. In such cases, the propagator method presented in this letter would be useful for finding explicit expression of light neutrino masses and collider signatures of bulk neutrinos [11] avoiding KK mode sums. The unified dynamics including charged fermions and intermediate-scale Majorana masses remains to be explored in future work.

Acknowledgments

This work is supported in part by the scientific grant from the ministry of education, science, sports, and culture of Japan (No. 20740135) and also by the grant-in-aid for the

global COE program "The next generation of physics, spun from universality and emergence" and the grant-in-aid for the scientific research on priority area (#441) "Progress in elementary particle physics of the 21st century through discoveries of Higgs boson and supersymmetry" (No. 16081209).

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